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Diffusion and creep of a particle in a random potential

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We investigate the diffusive motion of an overdamped classical particle in a 1D random potential using the mean first-passage time formalism and demonstrate the efficiency of this method in the investigation of the large-time dynamics of the particle. We determine the *log*-time diffusion $\langle\langle x^2(t)\rangle_{\rm th}\rangle_{\rm dis} = A\ln^\beta(t/t_r)$ and relate the prefactor A, the relaxation time t_r, and the exponent β to the details of the (generally non-gaussian) long-range correlated potential. Calculating the moments $\langle\langle t^n\rangle_{\rm th}\rangle_{\rm dis}$ of the first-passage time distribution $P(t)$, we reconstruct the large time distribution function itself and draw attention to the phenomenon of intermittency. The results can be easily interpreted in terms of the decay of metastable trapped states. In addition, we present a simple derivation of the mean velocity of a particle moving in a random potential in the presence of a constant external force.

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The motion of an overdamped classical particle in a random potential provides an effective description for a variety of phenomena, such as the dynamics of dislocations in solids and of domain walls in random field magnets (see Ref. 1 and references therein), the relaxation in glasses (see Ref. 2 and references therein), and the electrical transport in disordered solids (see Ref. 3 and references therein). Recently, this problem has been considered as a phenomenological model in the context of glassy dynamics of elastic manifolds in a quenched random medium[4–6]. In this paper we draw attention to the mean first-passage time formalism which appears to be very effective for the calculation of different characteristics of the random motion.

We start with the problem of particle diffusion in a long-range correlated random potential and generalize the results for the mean squared diffusion amplitude $\langle x^2(t)\rangle = A\ln^\beta(t/t_r)$ obtained in Refs. 7–9 to the case of nongaussian disorder, including the calculation of the exponent β and estimates for the prefactor A and the relaxation time t_r. Second, we investigate the motion of a particle subject to a random potential and driven by a constant external force. First, we shall briefly describe the mean first-passage time formalism.

Consider an overdamped particle in d dimensions subject to the potential $V(\mathbf{x})$ and a gaussian random force $\eta(t)$ with the correlator $\langle\eta_\alpha(t)\eta_\beta(t')\rangle = 2T\delta_{\alpha\beta}\delta(t-t')$, T denotes the temperature. The boundary conditions involve reflecting and absorbing walls S_r and S_a, respectively. The probability function $P(\mathbf{x},t)$ is obtained as the solution of the Fokker-Planck equation

$$\frac{\partial P}{\partial t} = D\frac{\partial^2 P}{\partial \mathbf{x}^2} + \frac{\partial}{\partial \mathbf{x}}\left(\frac{\partial V}{\partial \mathbf{x}}P\right), \tag{1}$$

with the boundary conditions $P(\mathbf{x}\in S_a)=0$ and $(\mathbf{n}\cdot\nabla)\,P(\mathbf{x}\in S_r)=0$, with $\mathbf{n}$ the unit vector perpendicular to the surface S_r. The diffusion constant D is related to the amplitude of the random force in the usual way, $D=T$ (we set the particle mobility equal to unity). Given the initial condition $P(\mathbf{x},t=0)=\delta(\mathbf{x}-\mathbf{y})$, the n-th moment $t_n(\mathbf{y}) = \langle t^n(\mathbf{y})\rangle_{\rm th}$ of the mean absorption time $t(\mathbf{y})$ satisfies the Pontryagin equation[10–12]

$$D\frac{\partial^2 t_n}{\partial \mathbf{y}^2} - \frac{\partial V}{\partial \mathbf{y}}\frac{\partial t_n}{\partial \mathbf{y}} = -n t_{n-1}(\mathbf{y}). \tag{2}$$

The above equation has to be solved in a closed region with the boundary conditions $t_n|_{S_a}=0$ and $(\mathbf{n}\cdot\nabla)\,t_n|_{S_r}=0$, see Refs. 10–12. Eq. (2) is equivalent to a chain of equations; taking into account that $t_0=1$ we can determine t_1 and proceeding by iteration we find all the moments of $t(\mathbf{y})$ and, consequently, can reconstruct the probability distribution function for $t(\mathbf{y})$. Let us apply this formalism to the problem of the 1D motion of a particle in a random environment.

First-passage time moments and distribution function. The equation of motion of a 1D overdamped classical particle moving in a random potential $U(x)$ takes the form

$$\dot{x} = -\frac{dU}{dx} + \eta(t). \tag{3}$$

The Fokker-Planck equation associated with the stochastic equation (3) is identical to Eq. (1), with $D = T$, $x \in \mathcal{R}^1$, and $V(x) = U(x)$. Let us consider the case where $U(x)$ is a gaussian random potential with correlator $\langle (U(x) - U(y))^2 \rangle_{\rm dis} = K(x - y)$, $K(u) \to C|u|^\alpha$, $u \to \infty$. The exponent α is assumed to be positive (if $\alpha < 0$, the disorder merely leads to a renormalization of the diffusion coefficient[12]). Assume that initially the particle is situated at the point $x = y$ of the interval $[0, L]$, with the boundaries 0 and L reflecting and L is absorbing the particle, respectively. Our aim is to calculate the disorder-averaged mean first-passage time. The discrete version of the above problem has been studied in a number of papers[13–17]. for the case of a random force with $\alpha = 1$.

For the case $d = 1$ Eq. (2) can be solved exactly, yielding (we remind the reader that y denotes the starting point of the particle's diffusion trajectory)

$$t_n(y) = \frac{n}{T} \int\limits_y^L dy_1 e^{U(y_1)/T} \int\limits_0^{y_1} e^{-U(x_0)/T} t_{n-1}(x_0)\, dx_0. \tag{4}$$

Expressing the solution for $t_{n-1}(y)$ in terms of $t_{n-2}(y)$, substituting into Eq. (4), and proceeding iteratively, we obtain the result

$$t_n(y) = \frac{n!}{T^n} \int_y^L dy_n \int_0^{y_n} dx_{n-1} \ldots \int_{x_1}^L dy_1 \int_0^{y_1} dx_0 \exp\left\{ \frac{1}{T} \left(\sum_{i=1}^n U(y_i) - \sum_{i=0}^{n-1} U(x_i) \right) \right\}. \tag{5}$$

After averaging over the gaussian disorder we arrive at the expression for the moment $\langle t_n(0) \rangle_{\rm dis}$

$$\langle t_n(0) \rangle_{\rm dis} = \frac{n!}{T^n} \int_0^L dy_n \int_0^{y_n} dx_{n-1} \ldots \int_{x_1}^L dy_1 \int_0^{y_1} dx_0 \\ \exp\left\{ \frac{\sum\limits_{i,j} [K(x_i - y_j) - K(x_i - x_j) - K(y_i - y_j)]}{2T^2} \right\}, \tag{6}$$

where Eq. (6) imposes the $2n$ restrictions

$$0 \le x_{i-1} \le y_i, \quad i = 1...n, \tag{7}$$

$$x_i \le y_i \le L, \quad i = 1...n. \tag{8}$$

In general, the integral in Eq. (6) cannot be calculated exactly. However, in the large distance L limit we can determine the integral by the method of steepest descents, thus describing the large-t diffusion. It can be easily seen that the integrand reaches its maximum at the point $(x_i, y_i) = (0, L)$ (note that the restrictions (7) and (8) are satisfied). As $K(0) = 0$ we obtain

$$\langle t_n(0) \rangle_{\rm dis} \sim \exp\left(\frac{n^2 K(L)}{2T^2} \right). \tag{9}$$

The prefactor is determined by the functional dependence close to the saddle-point. Expanding the expression in the exponent of (6) and taking into account that $K'(0) = 0$ we arrive at the final result for the n-th moment[18] of t

$$\langle t_n(0) \rangle_{\rm dis} = \frac{n!}{T^n} \left(\frac{2T^2}{K'(L) n} \right)^{2n} \exp\left(\frac{n^2 K(L)}{2T^2} \right). \tag{10}$$

The fact that the maximum of the integral in Eq. (6) is realized at the boundary manifests itself through the appearence of the first rather than second derivative of $K(x)$ in Eq. (10).

Using Eq. (10) we can reconstruct the tails of the probability distribution function for the first-passage time $t \equiv t(0)$. We look for a function of the form $P(t) \sim \exp\left(-A \ln^\gamma \left(t/\tilde{t}_0\right)\right)$. Calculating the moments of $P(t)$ by steepest descents[19] and comparing with Eq. (10) we find

$$P(t) \sim \exp\left(-\frac{T^2}{2K(L)} \ln^2 \left(t/\tilde{t}_0\right) \right), \tag{11}$$

where $\tilde{t}_0(T,L)$ is a microscopic time scale (while $\tilde{t}_0$ accounts for the (dimensional) prefactor in Eq. (10), its full dependence on L and T cannot be reconstructed from the large-t asymptotics alone). The asymptotic expression (11) is applicable for $t \gtrsim \tilde{t}_0$ and produces the strong intermittency observed in the moments $\langle t_n \rangle_{\rm dis}$ (see Eq. (10)). Note that the conjecture $\langle t_n(0)\rangle_{\rm dis} \sim (\langle t_1(0)\rangle_{\rm dis})^n$ made in Ref. 15 is inconsistent with our findings.

Large-time diffusion. Using the result (11) we can extract a lot of information concerning the large-t behavior of the particle. Suppose that at $t=0$ the particle is at the point $x=0$. Let us estimate its squared average displacement after a time t. The characteristic value of $x(t)$ can be found from the implicit equation $\left(T^2/K(x)\right)\ln^2\left(t/\tilde{t}_0\right) \sim 1$ (see Eq. (11)) defining the characteristic value of x where the probability distribution function $P(t)$ becomes negligible. With $K(x) \sim C|x|^\alpha$, we easily find that

$$\langle x^2(t)\rangle \sim \left(\frac{T^2}{C}\right)^{2/\alpha} \ln^{4/\alpha}(t/t_r)\,, \tag{12}$$

where t_r is a macroscopic diffusion or relaxation time. An estimate for t_r is obtained by comparing the characteristic barrier $U_L \sim \sqrt{CL^\alpha}$ on scale L with the temperature T. This defines the microscopic diffusion scale

$$L_T \sim \left(T^2/C\right)^{1/\alpha} \tag{13}$$

and its associated diffusion time

$$t_r \sim L_T^2/D \sim (1/T)\left(T^2/C\right)^{2/\alpha}. \tag{14}$$

Strictly speaking, the problem of the diffusion on a semi-axis which we consider here (the boundary $x=0$ is reflecting) differs from that of the diffusion on the whole axis. However, the boundary condition at $x=0$ affects the answer (see Eq. (12)) only by a factor of order unity.

Eq. (11) can be interpreted in terms of the decay of metastable trapped states. Suppose that a particle leaves a metastable state via thermal activation over a barrier of random height. Let the barrier distribution function be gaussian[20], $P(U) = \left(1/\sqrt{2\pi}\Delta\right)\exp\left(-U^2/2\Delta^2\right)$. Then the probability distribution of lifetimes $\tilde{t}_0 \exp(U/T)$ is given by the expression (we assume that $t > \tilde{t}_0$, see also Ref. 20)

$$P(t) = \int\limits_0^{+\infty} \frac{dU}{\sqrt{2\pi}\Delta}\exp\left(-\frac{U^2}{2\Delta^2}\right)\delta\left(t-\tilde{t}_0 e^{U/T}\right) = \frac{T}{\sqrt{2\pi}\Delta t}\exp\left\{-\frac{T^2}{2\Delta^2}\ln^2\left(t/\tilde{t}_0\right)\right\}. \tag{15}$$

Eq. (15) exhibits the same large-t dependence as Eq. (11), implying that the diffusion on the scale L is dominated by one deep potential well of characteristic depth $K^{1/2}(L)$. This interesting feature of 1D diffusion allows us to generalize Eq. (12) for the case of non-gaussian disorder.

Assume that the probability for the potential difference $U(x+L)-U(x)$ to be equal to E is given by the function

$$P(E) = \frac{\delta}{2^{1+1/\delta}K^{1/\delta}(L)\Gamma(1/\delta)}\exp\left\{-\frac{|E|^\delta}{2K(L)}\right\}, \tag{16}$$

with $K(L) \to CL^\alpha, L \to \infty$. Calculating $P(t)$ in the same way as in Eq. (15) above, we arrive at the result (see also Ref. 20)

$$P(t) \sim \exp\left\{-\frac{T^\delta|\ln\left(t/\tilde{t}_0\right)|^\delta}{2K(L)}\right\}, \tag{17}$$

implying a $\log t$-diffusion of the form

$$\langle x^2(t)\rangle \sim \left(\frac{T^\delta}{C}\right)^{2/\alpha}\ln^{2\delta/\alpha}(t/t_r)\,, \tag{18}$$

where $t_r \sim (1/T)\left(T^\delta/C\right)^{2/\alpha}$ is the relaxation time for non-gaussian disorder. One can easily verify that Eq. (17) implies that $\ln\langle t_n\rangle_{\rm dis} \sim n^{\delta/(\delta-1)}$. For $\delta \to \infty$, the barriers in the system vanish and the phenomenon of intermittency disappears.

Eq. (18) is consistent with the exactly solvable gaussian random force problem, for which the amplitude before the logarithm $A = (61/45)\,T^4/C^2$ is known exactly and the relaxation time $t_r \sim T^3/C^2$, see Refs. 1,7,8. For the case

of gaussian disorder ($\delta = 2$) the exponent $4/\alpha$ of the logarithm in Eq. (18) has been obtained using RG-techiques[9]. The mean first-passage time method allows us to estimate the amplitudes and characteristic relaxation times and to generalize the results to the case of non-gaussian disorder. Furthermore, Eq. (17) generalizes the results of previous investigations of the gaussian random force model[17] to the case of arbitrary disorder.

Creep under the action of an external force. Let us apply the mean first-passage time formalism to the problem of creep in 1D. In a recent paper[4], Le Doussal and Vinokur reported results on the mobility of an overdamped classical particle moving in a 1D random potential $U(x)$ in the presence of a constant external force f (see also Ref. 5). The mean velocity V has been calculated as a function of f and the correlator $K(x)$ of the random potential. The above model has been considered as a phenomenological model of glassy dynamics: It turns out that long-range correlations of the random potential lead to the glassy response $V \sim \exp\left(-1/Tf^{\mu}\right)$ as $f \to 0$, with T the temperature and $\mu > 0$ a constant, whereas in the case of short-range correlations $V \sim f$ as $f \to 0$.

The problem has been solved[4] using a generalization of the method introduced by Derrida[21] for discrete models, assuming that the random potential is a periodic function of the coordinate x, $U(x) = U(x + L)$. Next, the stationary solution $\tilde{P}(x)$ of the Fokker-Planck equation has been found for a fixed current $\tilde{J}$. Using the conditions $\tilde{P}(0) = \tilde{P}(L)$, $U(0) = U(L)$, $V = \tilde{J}L$, the normalization condition on $\tilde{P}(x)$, and taking the limit $L \to \infty$, the authors of Ref. 4 arrive at the result (see also Ref. 5)

$$\frac{1}{V} = \frac{1}{T}\int_0^{\infty} ds e^{-fs/T}\langle e^{(U(x+s)-U(x))/T}\rangle_x, \tag{19}$$

where

$$\langle A\rangle_x = (1/L)\lim_{L\to\infty}\int_0^{L} dx A(x). \tag{20}$$

For the case of a gaussian random potential with $\langle (U(x) - U(y))^2\rangle_{\text{dis}} = K(x - y)$ the averaging procedure can be easily performed, yielding the final result

$$\frac{1}{V} = \frac{1}{T}\int_0^{\infty} dx \exp\left(-\frac{fx}{T} + \frac{K(x)}{2T^2}\right). \tag{21}$$

Let us show how Eqs. (19) and (21) can be obtained in a simple and elegant way using the first-passage time method. This technique has several advantages: *i*) it does not rely on the periodic continuation of the random potential and, consequently, one does not have to worry about the commutation of the two limits $t \to \infty$ and $L \to \infty$; *ii*) instead of solving the stationary Fokker-Planck equation with a fixed current one can use the well-known solutions of the 1D Pontryagin equation[10–12] and simply average them over the disorder; *iii*) using the first-passage time technique one can investigate the finite size effects, the moments of the average velocity distributon function etc., i.e., the information obtained is much richer.

Returning to the 1D problem of an overdamped particle subject to the potential $V(x) = U(x) - fx$ we can solve Eq. (2) for the first moment exactly[10–12],

$$t_1(y) = \frac{1}{T}\int_y^{L} dz\, e^{V(z)/T}\int_a^{z} dx\, e^{-V(x)/T}. \tag{22}$$

Here we assume that the point L is absorbing, the point a is reflecting[22], and $a < y < L$. In the limit $a \to -\infty$ the particle does not feel the left boundary as the external force f is chosen to be positive. Averaging Eq. (1) over the disorder we obtain

$$\langle t_1(y)\rangle_{dis} = \frac{1}{T}\int_y^{L} dz\, e^{-fz/T}\int_{-\infty}^{z} dx\, e^{fx/T}\langle e^{(U(z)-U(x))/T}\rangle_x. \tag{23}$$

If the random potential distribution is spacially homogeneous, the average $\langle e^{[U(z)-U(x)]/T}\rangle_{\text{dis}}$ is a function of $|z - x|$ and is the same as the translation average $\langle e^{[U(x+s)-U(x)]/T}\rangle_x$ (see Eq. (19)), where we have introduced the new variable $s = x - z$. Eq. (23) then takes the form

$$\langle t_1(y)\rangle_{\rm dis} = \frac{L-y}{T}\int\limits_0^\infty ds\, e^{-fs/T}\langle e^{(U(x+s)-U(x))/T}\rangle_{\rm dis}. \tag{24}$$

If the integral in Eq. (24) converges, the mean first passage time averaged over disorder is an extensive quantity. In the limit $L-y\to\infty$ we can apply the central limit theorem, implying that $1/\langle t_1(y)\rangle_{\rm dis} = \langle 1/t_1(y)\rangle_{\rm dis}$. The mean velocity V is simply $\langle (L-y)/t_1(y)\rangle_{\rm dis}$ and we can easily see that Eq. (24) becomes equivalent to Eq. (19). For a gaussian random potential,

$$\langle t_1(y)\rangle_{\rm dis} = \frac{L-y}{T}\int\limits_0^\infty ds\, \exp\left(-\frac{fs}{T}+\frac{K(s)}{2T^2}\right), \tag{25}$$

which is equivalent to Eq. (21).

Relation to discrete models. Let us compare the mean first passage-time averaged over disorder obtained using Eq. (22) with that found for a discrete 1D random walk in a random force-field. Consider a 1D disordered lattice. Let us denote by p_n the probability of hopping forward $n\to n+1$ and by $q_n = 1-p_n$ that of hopping back $n\to n-1$. The index n enumerates the sites of the lattice. For the random-force problem[1],

$$p_n = \frac{\exp\left[\frac{dF_{n+1}}{2T}\right]}{\exp\left[-\frac{dF_n}{2T}\right]+\exp\left[\frac{dF_{n+1}}{2T}\right]}, \tag{26}$$

with d the lattice spacing (note that in this model *both* time and space are discrete). The force F is a gaussian random variable satisfying the conditions $\langle F_n F_m\rangle = \mu\delta_{nm}/d$ and $\langle F_n\rangle = 0$. Note that $\langle \ln(p_n/q_n)\rangle = 0$, see Refs. 1,7. The point $n=0$ is supposed to be reflecting and $N = L/d$ is absorbing. In Ref. 15 the estimate $\langle t_1(0)\rangle_{\rm dis} \sim \exp[\beta L]$, where $\beta = \ln\langle q_n/p_n\rangle = \mu/4T^2$ and $L\to\infty$, has been obtained.

The continuous version of this problem is described by Eq. (3) with a correlator for the random potential given by $K(x-y) = \mu|x-y|$. Averaging Eq. (22) over disorder we obtain

$$\langle t_1(y)\rangle_{\rm dis} = \frac{4T^3}{\mu^2}\left[\exp\left(\frac{\mu L}{2T^2}\right)-\exp\left(\frac{\mu y}{2T^2}\right)\right]-\frac{2T}{\mu}\left(L-y\right). \tag{27}$$

For y close to the reflecting boundary 0 and for large L the result takes the simple form

$$\langle t^{(1)}(y)\rangle_{\rm dis} \to \frac{4T^3}{\mu^2}\exp\left(\frac{\mu L}{2T^2}\right). \tag{28}$$

We find that for both (discrete in space and time and continuous) cases $\ln\langle t_1(0)\rangle_{\rm dis} \sim L$. The coefficient of proportionality, however, is different, $\beta = \mu/2T^2$ for the continous model versus $\beta = \mu/4T^2$ for the discrete case.

We thus have arrived at a different asymptotic behavior for the average first-passage time. The origin of this difference is found in the inequivalence of the discrete in space and time and the discrete in space–continuous time models: For the latter, consider the lattice master equation describing the probability $P_n(t)$ for the particle to be on the site n:

$$\frac{dP_n}{dt} = W_{n,n+1}P_{n+1} + W_{n,n-1}P_{n-1} - \left(W_{n+1,n}+W_{n-1,n}\right)P_n, \tag{29}$$

where the hopping probabilities $W_{n,n\pm1}$ and $W_{n\pm1,n}$ are determined by the potential[1] (as the lattice spacing tends to zero, one easily recovers the continuous Fokker-Planck equation (1) and thus this model is equivalent to the continuous in space and time model). The terms $W_{n,n+1}P_{n+1}$ and $W_{n,n-1}P_{n-1}$ in Eq. (29) describe the hopping $n+1\to n$ and $n-1\to n$, respectively. The contribution $\left(W_{n+1,n}+W_{n-1,n}\right)P_n$ describes the probability to stay at the same site n. In a next step let us also discretize the time variable t. Suppose that at time t, $P_n = \delta_{nm}$. In the limit of a finite but small time step Δt the probabilities for the processes. $m\to m\pm1$ behave as $\alpha_1\Delta t$ and $\alpha_2\Delta t$ (α_1 and α_2 are two constants), hence the probability for a particle to stay at the site m after $t\to t+\Delta t$ is $1-(\alpha_1+\alpha_2)\Delta t$, i.e. the particle most likely stays at the same site. On the other hand, for the discrete lattice walks considered in Refs. 13,15–17 the probability of staying at the same site vanishes by definition: The particle may only jump to the neighbouring sites. Thus the continuous random walk cannot be obtained as a limiting case of a discrete random walk when *both* the time and the lattice are discrete.

Briefly summarizing, we have investigated the motion of an overdamped classical particle in a random potential. The moments of the first-passage time averaged over gaussian disorder have been found and are given by Eq. (10). The asymptotic form of the first-passage time distribution function has been reconstructed, see Eq. (11), allowing us to find the large-t dependence of $\langle x^2(t)\rangle$, see Eq. (12). The results obtained can be easily interpreted in terms of the decay of metastable states. The latter feature has allowed us to generalize the results for $\langle x^2(t)\rangle$ to the case of non-gaussian disorder, see Eq. (18). In addition, we have presented a simple derivation of the mean velocity of a particle driven by a constant external force across a disordered medium and have discussed the relation to the discrete time random walk on a lattice.

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[18] We wish to point out that the first moment $\langle t_1\rangle \sim \exp\left(K(L)/2T^2\right)$ determines the *average* first-passage time, while the *typical* time t_{typ} of the first-passage which controls the large time diffusion scales as $\ln t_{\mathrm{typ}} \sim \sqrt{K(L)}/T$, see also Refs. 13,15.
[19] The function $P(t)$ should satisfy the conditions: *i)* $P(t)$ is continuous; *ii)* tends to zero sufficiently fast such that all its moments exist for arbitrary n. The reconstruction of the probability distribution function $P(t)$ involves the inverse Mellin transformation: Let $\langle t_n\rangle = \int_0^\infty t^n P(t)dt$ be a regular function of the variable $n = \sigma + i\tau$ in the strip $\sigma_1 < \sigma < \sigma_2$ and $\int_{-\infty}^{+\infty} |\langle t_{\sigma+i\tau}\rangle| d\tau < \infty$, then $P(t) = (1/2\pi i)\int_{c-i\infty}^{c+i\infty} t^{-n-1}\langle t_n\rangle\, dn$, with $\sigma_1 < c < \sigma_2$. For the function $\langle t_n\rangle$ given by Eq. (10) the main term in the asymptotics for $P(t\to\infty)$ is determined by the exponential $\exp\left(n^2K(L)/2T^2\right)$. The integral in the inversion formula then can be calculated by steepest descents.
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